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Onset of natural convection in a rotating fluid layer with non-uniform volumetric heat sources

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Abstract

A study of the thermal instability in an initially quiescent liquid, placed between two horizontal plates, irradiated by a volumetric heat source, and subject to rotation about the vertical axis is carried out. The eigenvalue problem is solved using Chebyshev pseudospectral methods employing the basis recombination technique for higher order problems. The effect of the orientation of the heat source, as well as boundary conditions on the onset of natural convection is studied for different rotation rates. Rotation is seen to unconditionally increase the stationary stability in all cases. Our results indicate a linear relationship between critical Rayleigh number and Taylor number at moderate values of Taylor number, and also show that convection in rotating fluid layer is more sensitive to the exact nature of the heat source distribution in case of stable and quasi stable temperature profiles.

Keywords: Convection; Stability; Rotation; Volumetric heat generation; Pseudospectral methods

1. Introduction

Thermal convection in rotating, internally heated fluid layers is of great interest in the microwave heating of liquids [1,2]. The most important and widespread application of microwave heating is in the domestic microwave oven. Microwave heating also finds application in the food industry such as in pasteurization, sterilization etc. A study of internally heated, rotating convection processes is also relevant in understanding the dynamics of atmospheric convection in the stars and the major planets [3].

Natural convection due to internal volumetric heat sources has been studied both experimentally [4,5] and numerically [6–12]. Sparrow et al. [6] investigated the generic case of thermal instabilities in a fluid layer with non-linear temperature profile. Thermal stability in the case of uniform volumetric heat source was studied by Kulacki and Goldstein [7] for a range of boundary conditions using both the linear stability theory and the energy stability theory. Stability of convection due to non-uniform volumetric heat sources was studied by Yucel and Bayazitoglu [8] and they considered an exponentially decaying heat source generation. They also carried out similar investigations [9] on fluid layers with free surfaces. Sorour and Hassab [10] studied transient thermal stability in fluid layers with non-uniform heat sources. Stability with non-uniform heat sources as well as unequal surface temperatures have been studied by Hassab [11].

In the earlier literature [4–11], the expression for power source intensity as well as boundary conditions were quite complicated and the explicit effects of heat source distributions were not studied in detail. A study on the effect of heat source distributions in convection with a special emphasis on exponential heat sources was done by Tasaka and Takeda [12]. However, the system in this case was stationary. The present study extends this work [12] to the case of rotating fluid layers and also investigates the effect of various thermal boundary conditions.

The effect of rotation in Rayleigh Benard convection was one of the earliest interests of Chandrasekhar [13,14]. Chandrasekhar studied the effect of rotation in a series of papers [13,14] and the results are described in detail on his comprehensive treatise for hydrodynamic stability [15]. However, Chan-

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Nomenclature

γ	horizontal wavenumber
γ_x	wave number in the x-direction
γ_y	wave number in the y-direction
c_p	specific heat
D	derivative with respect to z-direction
$\mathbf{g}$	acceleration due to gravity vector
L	height of fluid layer
N	number of collocation points
p	pressure
Pr	Prandtl number
Q	volumetric heat source
R	Rayleigh number
T	temperature
ΔT	temperature difference between the maximum and minimum temperatures occurring in a distribution
Ta	Taylor number
t	time
$\mathbf{u}$	velocity vector
w	velocity in the z-direction
z	co-ordinate in the vertical direction
z'	height at which maximum temperature is reached

Greek symbols

β	bulk modulus
ε, η	characteristic length of the heat source distribution
θ	Chebyshev polynomial
κ	thermal diffusivity
λ	thermal conductivity
μ	viscosity of fluid
ξ	vertical component of vorticity
ρ	density
σ	temporal growth rate of perturbations
$\mathbf{\Omega}$	rotation vector
Ω	speed of rotation
ψ	trial function

Subscripts

j	jth polynomial
0	properties at $T^* = T_1$
C	critical value
I	internal

Superscripts

$*$	dimensional variable
$\bar{\ }$	stationary value
$\hat{\ }$	separated variable in perturbed value which is a function of z
$'$	perturbed value

drasekhar's investigation was confined to classical Rayleigh Benard convection without any volumetric heat generation. Veronis [16] studied cellular patterns in rotating convection. Rotating convection in two and three dimensions were studied numerically using Galerkin methods by Clever and Busse [17].

A significant amount of literature has been devoted on the stability of convection due to internal heat sources. A limited number of works [18,19] on stability of horizontal fluid layers under the dual effect of rotation and internal heat generation are also available in the literature. Although earlier works [18,19] deal with convection in rapidly rotating spheres with uniform internal heating, but the problem of convection in rotating systems with spatially non-uniform heat sources has been overlooked. One of the reasons may be because of the numerical complications involved, since, as it shall be seen later, the resulting differential equation, with the inclusion of rotation, becomes eighth order, and hence a numerical solution to the problem becomes increasingly difficult. A remark may be made here that studies on the dual effect of rotation and internal heat generation are important as the momentum and energy balance equations are coupled and the effect of the two factors cannot be independent on each other.

The objective of the present investigation is to analyze the stability criteria of a horizontal fluid layer undergoing rotation, irradiated by an internal heat source. We consider various orientations of the heat source and their effects on the onset of cellular convection in the rotating horizontal layer subject to different boundary conditions. The prime interest of the present work is on applications such as microwave heating with exponential heat sources. Convection due to microwave heating with top and bottom incidences may be a classic example of internal heat sources as studied by earlier researchers [20,21]. It may be noted that microwave heat sources, for the special case of large depths, behave as an exponentially decaying heat source. This formulation of the microwave heat source is known as the Lambert's law and has been applied in studying microwave heating in many cases [1,22,23]. In fact, Lambert's law is valid for samples thicker than three times the characteristic depth [24].

2. Problem formulation

The flow in a horizontal fluid layer of height L kept between two parallel plates is considered as shown in Fig. 1. The fluid layer is undergoing rotation about the z-axis and is irradiated by an internal heat source $Q(z^*)$. The fluid is enclosed by rigid boundaries, both at the top and the bottom. Two cases of thermal boundary conditions have been considered. Initially, we investigate the condition when the upper boundary is at a constant temperature and the lower boundary is adiabatic. The other situation with the upper and the lower boundaries at the same constant temperature T_1 has also been studied. The schematic is shown in Fig. 1. The governing equations are

$$\nabla \mathbf{u}^* = 0 \tag{1}$$

$$\rho_0\left(\frac{\partial \mathbf{u}^*}{\partial t^*} + (\mathbf{u}^*.\nabla)\mathbf{u}^*\right) = -\nabla\left(p^* - 1/2\rho_0(\mathbf{\Omega}\times\mathbf{r}^*)^2\right) - 2\rho_0\mathbf{\Omega}\times\mathbf{u}^* - \rho\mathbf{g} + \mu\Delta\mathbf{u}^* \tag{2}$$

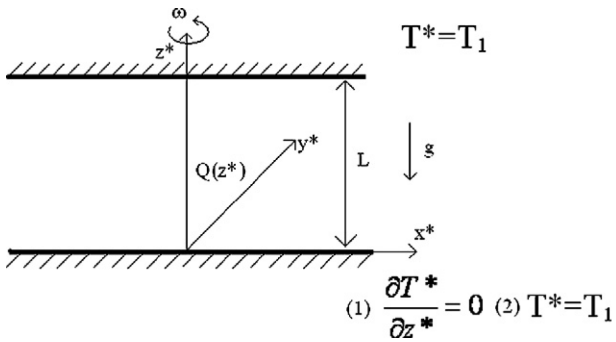


Fig. 1. Schematic of the physical system with two sets of boundary conditions at the bottom:adiabatic and isothermal.

$$\rho_0 c_p\left[\frac{\partial T^*}{\partial t^*}+(\mathrm{u}^*.\nabla)T^*\right]=\lambda\Delta T^*+Q(z^*) \tag{3}$$

where $\Delta=\frac{\partial}{\partial x^{*2}}+\frac{\partial}{\partial y^{*2}}+\frac{\partial}{\partial z^{*2}}$.

The no slip boundary conditions have been applied at the top and the bottom wall.

The physical quantities have been non-dimensionalized in the usual way. It may be noted that the non-dimensional temperature is expressed as $T=\frac{T^*}{\Delta T}$, where ΔT is a function of the power source distribution (and of the boundary conditions). Consistency has been maintained by defining ΔT as the maximum temperature difference obtained in the base state for the particular heat source distribution considered.

In the basic state, the system is static, i.e. $\mathbf{u}=\mathbf{0}$ and the temperature distributions for the various arrangements of heat sources can be obtained by solving the simple differential equation

$$\lambda\frac{\partial^2 T^*}{\partial z^{*2}}=-Q(z^*) \tag{4}$$

The study the stability of this flow is carried out using the method of small linear perturbations.

Performing normal mode expansion of each small perturbation in the form:

$$\begin{bmatrix}u'\\p'\\T'\end{bmatrix}=\begin{bmatrix}\hat{u}(z)\\\hat{p}(z)\\\hat{T}(z)\end{bmatrix}\exp\bigl(\mathrm{i}(\gamma_x x+\gamma_y y)+\sigma t\bigr) \tag{5}$$

The following perturbation equations have been arrived at:

$$(D^2-\gamma^2-\sigma)\hat{T}=-\frac{\partial\bar{T}}{\partial z}\hat{w} \tag{6}$$

$$\left(D^2-\gamma^2-\frac{\sigma}{Pr}\right)\hat{\xi}=-\sqrt{Ta}\,D\hat{w} \tag{7}$$

$$\left(D^2-\gamma^2-\frac{\sigma}{Pr}\right)(D^2-\gamma^2)\hat{w}-\sqrt{Ta}\,D\hat{\xi}=R_I k^2\hat{T} \tag{8}$$

where $D\equiv\frac{\partial}{\partial z}$ and $\gamma=\sqrt{\gamma_x^2+\gamma_y^2}$.

The major non-dimensional parameters are the Prandtl number (Pr), the Taylor number (Ta), and the Rayleigh number R_I, where:

$$Pr=\frac{\nu}{\kappa_0}$$

$$Ta=\frac{4\Omega^2}{\nu^2}L^4$$

and R_I is the internal Rayleigh number whose definition depends on the configuration of the heat source distribution.

The hydrodynamic boundary conditions are:

$$\hat{w}=D\hat{w}=\hat{\xi}=0 \quad \text{at } z=0 \text{ and } 1 \tag{9}$$

and the thermal boundary conditions are:

(a)

$$\hat{T}=0 \quad \text{at } z=1 \tag{10}$$

$$D\hat{T}=0 \quad \text{at } z=0 \tag{11}$$

(b)

$$\hat{T}=0 \quad \text{at } z=0 \text{ and } 1 \tag{12}$$

The following assumption is made for the analysis. In the case of convection with rotation, the principle of exchange of stabilities does not hold in general, and as Chandrasekhar [15] remarked, there is no simple analytical criterion to determine when the principle holds. We shall restrict ourselves to the case when the principle holds, that is, when convection proceeds through a stationary instability. Hence, with $\sigma=0$, using the principle of exchange of stabilities, we arrive at the following final equation, eliminating both $\hat{T}$ and $\hat{\xi}$:

$$(D^2-\gamma^2)^3\hat{w}+Ta\,D^2\hat{w}=-R_I k^2\frac{\partial\bar{T}}{\partial z}\hat{w} \tag{13}$$

This equation is solved with boundary conditions (Eqs. (9)–(12)).

3. Method of solution

The equations constitute a sixth order system, but as Chandrasekhar [15] points out, since it involves boundary conditions involving $\hat{\xi}$ (Eq. (9)), the system is effectively eighth order. The numerical approach used in solving the above system is based on the pseudospectral discretizations in Chebyshev polynomials [25]. Each variable is expanded in terms of "trial functions". For example, we can write $\hat{w}$ as:

$$\hat{w}=\sum a_j\Psi_j \tag{14}$$

In order to make the system symmetric about $z=0$, the coordinates have been shifted to make the solution domain from $z=-0.5$ to $z=0.5$.

Since, in case of higher order problems, the derivatives of the Chebyshev polynomials oscillate near the end point with high amplitude, the basis recombination technique is employed, choosing the trial functions as suggested by Boyd [25], using the "Heinrichs" basis.

Using the "Heinrichs" basis, the trial functions for $\hat{w}$ have been chosen as:

$$\psi_j=(0.25-z^2)^2\theta_j \tag{15}$$

It can be clearly seen that these trial functions, obtained by basis recombination automatically satisfy the homogeneous boundary conditions for $\hat{w}$ (Eq. (9)) in the shifted co-ordinate system.

The trial function for $\hat{\xi}$ and for $\hat{T}$ (when the boundary condition is that of constant temperature both above and below) is

$$\psi_j = (0.25 - z^2)\theta_j \tag{16}$$

In the adiabatic case, the trial function for $\hat{T}$ becomes

$$\psi_j = (0.5 - z)\theta_j \tag{17}$$

and the adiabatic boundary condition is imposed numerically. Discretizing the variables in terms of trial functions, the solutions are obtained for N collocation points. This results in an $N \times N$ matrix eigenvalue problem of the form

$$\mathbf{AX} = R_I \mathbf{BX} \tag{18}$$

the solution of which yields the critical Rayleigh number.

This eigenvalue problem is solved by the usual Q–Z algorithm. The complete details of the numerical procedure to construct and solve the above problem can be found in Boyd [25]. Convergence of the solution was investigated by varying the number of collocation points. A change of N beyond 25 does not produce significant changes in the critical Rayleigh number. Hence, N was fixed at 25.

4. Validation

In order to establish the accuracy of the Chebyshev pseudo-spectral scheme, comparison with the results of previous studies were made. In the case of classical Rayleigh Benard convection with rotation, our scheme predicted results which are in excellent agreement with those of Chandrasekhar [15]. The comparison of the results is listed in Table 1.

The present scheme has also been validated with earlier work by Tasaka and Takeda [12] who investigated the onset of convection in horizontal fluid layers irradiated by exponential heat sources. In case of uniform volumetric heat sources, our scheme predicts the value of the critical Rayleigh number as 1378.34, which is quite close to the value ($R_{IC} = 1386.14$) predicted by Roberts [26] (the definition for the internal Rayleigh number by Roberts [26] is twice that of current work and hence the value was 2772.8 [26] in an identical study). The agreement of these results substantiates the applicability of the Chebyshev pseudo spectral method for determining the conditions leading to the onset of convective motions in a liquid layer.

Table 1
Comparison of the present work with Chandrasekhar [15]

Taylor number	Chandrasekhar		Present work	
	γ_C	R_{IC}	γ_C	R_{IC}
10	3.1	1713	3.1	1711.9
100	3.15	1756.6	3.15	1756.4
500	3.3	1940.5	3.3	1939.9
1000	3.5	2151.7	3.5	2150.6
2000	3.75	2530.5	3.75	2529.4
5000	4.25	3468.6	4.25	3463.8
10 000	4.8	4713.1	4.79	4712.1
30 000	5.8	8326.4	5.78	8324.5
100 000	7.2	16 721	7.18	16 716

γ_C represents the critical wave number and R_{IC} the critical Rayleigh number.

5. Results and discussion

5.1. Adiabatic boundary conditions

The effect of rotation is investigated mainly in three cases of heat source orientation:

1. Uniform heat source i.e. $Q = Q_0$.
2. An exponentially decaying heat source applied from the bottom boundary

$$Q = \frac{Q_0}{H(\varepsilon)} \exp\left(\frac{-z}{\varepsilon}\right) \tag{19}$$

where

$$H(\varepsilon) = \int_0^1 \exp\left(\frac{-z}{\varepsilon}\right) dz = \varepsilon\left[1 - \exp\left(\frac{-1}{\varepsilon}\right)\right] \tag{20}$$

Here ε is the characteristic length of the heat source distribution when applied from below. A decrease in ε implies the concentration of heat source at the bottom boundary.

Maintaining consistency with Eq. (13), the internal Rayleigh number in this case is defined as in Tasaka and Takeda [12],

$$R_I = \frac{g\beta Q_0 L^5}{\lambda \nu_0 \kappa_0} \frac{\varepsilon^2}{H(\varepsilon)}\left[\exp\left(-\frac{1}{\varepsilon}\right) + \frac{1}{\varepsilon} - 1\right]$$

3. An exponentially decaying heat source applied from the top boundary

$$Q = \frac{Q_0}{H(\eta)} \exp\left(\frac{z-1}{\eta}\right) \tag{21}$$

where

$$H(\eta) = \int_0^1 \exp\left(\frac{z-1}{\eta}\right) dz = \eta\left(1 - \exp\left(-\frac{1}{\eta}\right)\right) \tag{22}$$

η denotes the characteristic length of the heat source distribution when applied from above, and a decreasing η implies increased concentration of the heat source at the top boundary. In this case, the internal Rayleigh number is defined as [12]:

$$R_I = \frac{g\beta Q_0 L^5}{\lambda \nu_0 \kappa_0} \frac{\eta^2}{H(\eta)}\left[1 - \left(1 + \frac{1}{\eta}\right)\exp\left(-\frac{1}{\eta}\right)\right]$$

These configurations have been studied in detail for the stationary case by Tasaka and Takeda [12], and for a detailed discussion on the temperature profiles, perturbation equations on temperature and other details, the reader is directed to the above reference. Our main interest is to study the effect of rotation on the stability in these heat source configurations.

The variation of critical Rayleigh number with ε or η is shown in Fig. 2. Trends observed for the variation of R_{IC} with ε are similar to the static case, with the Rayleigh number proceeding asymptotically towards its critical value in the case of the uniform heat flux. When the heat flux is applied from above (variation with η) the sharp asymptotic variation is seen in all cases, and seems to increase with increasing Taylor number. This is further elaborated in the next section.

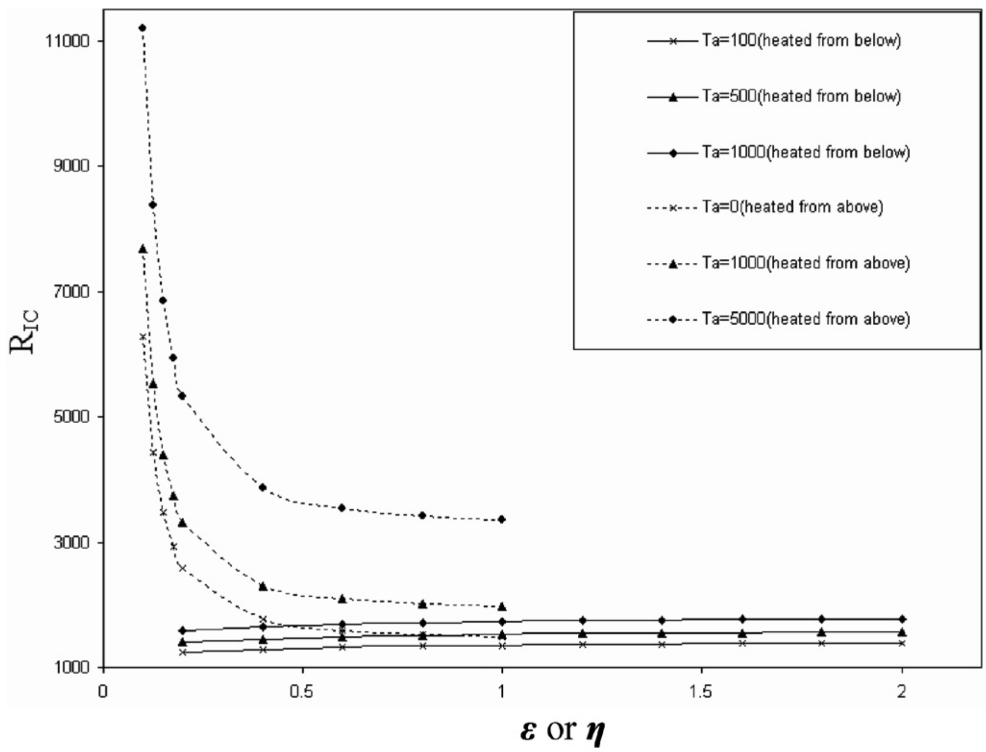


Fig. 2. The variation of critical Rayleigh number with ε or η for different values of Taylor number. The variation with ε is for the case when the heat source is applied from below, and the variation with η is for the case when the heat source is applied from above.

5.1.1. *Linear R_I–Ta relationship*

The effect of heat source distribution on the evolution of Rayleigh number with Taylor number has been investigated. Chandrasekhar [15] showed that at the asymptotic limit, the Rayleigh number scales as $Ta^{2/3}$. We have however, confined our investigations to moderate Taylor numbers, and find that in this range, the R_{IC}–Ta curve appears as a straight line as seen in Fig. 3. The linearity trend is unaffected by the orientation of the heat source distribution ,the value of the exponents and the nature of boundary conditions. Thus these detailed analyses help us to arrive at a very important conclusion, that the R_{IC}–Ta profile for moderate Taylor numbers is linear and this trend is independent of the nature of the heat source distribution and boundary conditions.

5.1.2. *Effect of rotation*

The slope of the R_I–Ta curve gives us important information on how rotation affects the stability of the system in various heat source configurations. Fig. 4 illustrates the slope of the R_{IC}–Ta curve as a function of the exponent (ε or η). It is interesting to note that the slope of the curve has only minor variation with change in ε, in the case of exponential heating from the bottom. Also, this slope is quite close to the value for the case of uniform heat distribution. However, when the exponential heat source is applied at the top, there is a distinct change in slope, depending on the value of η. The slope is larger for smaller values of η, and slope decreases asymptotically as η increases, towards the uniform heat distribution case. The steeper slopes at low η clearly indicate that rotation has a stronger effect on the stability of convection at low η.

The influence of rotation on temperature profiles has been discussed next. The temperature profile for both the cases is shown in Fig. 5. While the heat source at the bottom produces a distribution, which has a small negative slope for all values of ε, the concentration of heat source at the top is seen to give rise to a large region where the temperature is nearly constant. This zone leads to a high degree of stability, and has been termed as the “quasi-stable layer” by Tasaka and Takeda [12]. It may be noted that rotation leads to a greater stability of the fluid layer, and the quasi stable zone is seen to enhance this stability. Thus as the temperature gradient of the quasi-stable layer decreases, it enhances the rotation induced stability to a greater extent, leading to steeper slopes.

It may thus be concluded that in the absence of quasi stable layers, the stability of the rotating system is not very sensitive to the exact nature of the heat source distribution. Rotational effects become highly sensitive to the temperature profile, only when layers with very low temperature gradients are present. This is further elaborated in the next section.

Figs. 6 (a) and (b) show the variation of critical wave number with the rate of rotation. The critical wave number is seen to increase with rotation rates, indicating an increase in the size of the convection cells with rotation. However, the change in critical wave number with the configuration of the heat source distribution is similar to the static case in Tasaka and Takeda [12]. When the heat source is applied from the bottom, the wave number remains close to its value for an uniform heat source distribution (within 2% in the static case) and low values of η produce a significant deviation (about 14% in the static case) when the heat source concentration is at the top.

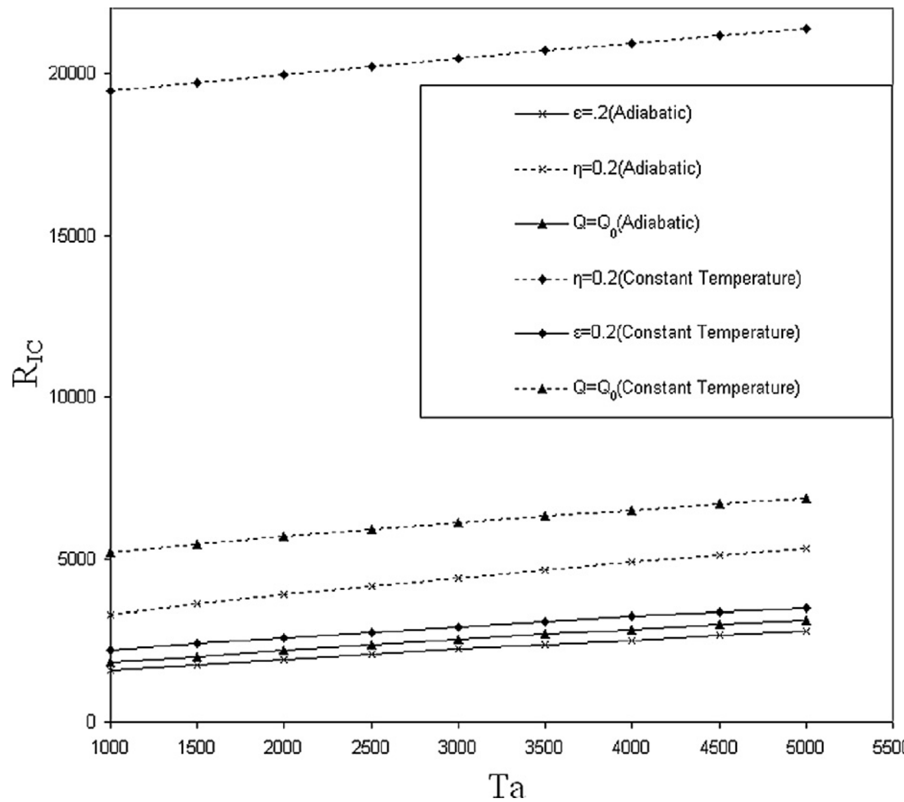


Fig. 3. Linear R_I–Ta relationship for moderate Taylor numbers (1000–5000), for different orientations of the heat source, both for the adiabatic and the constant temperature boundary conditions.

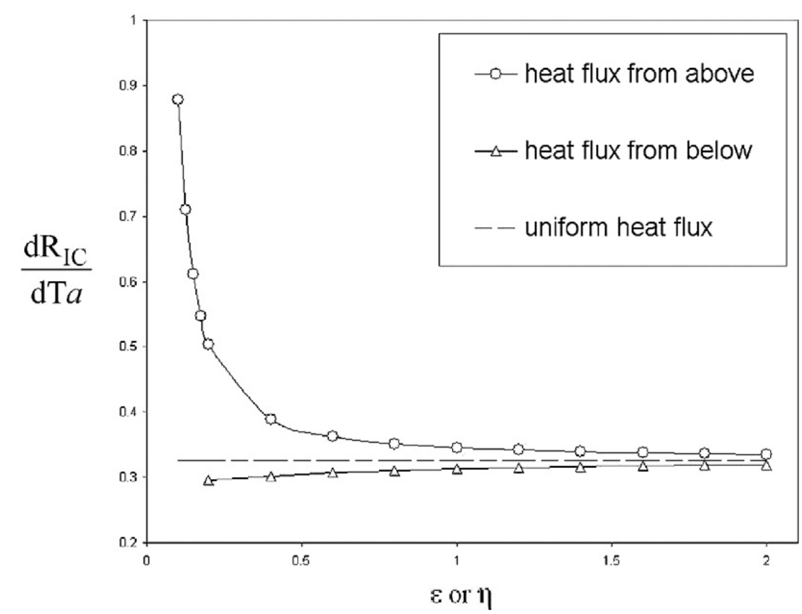
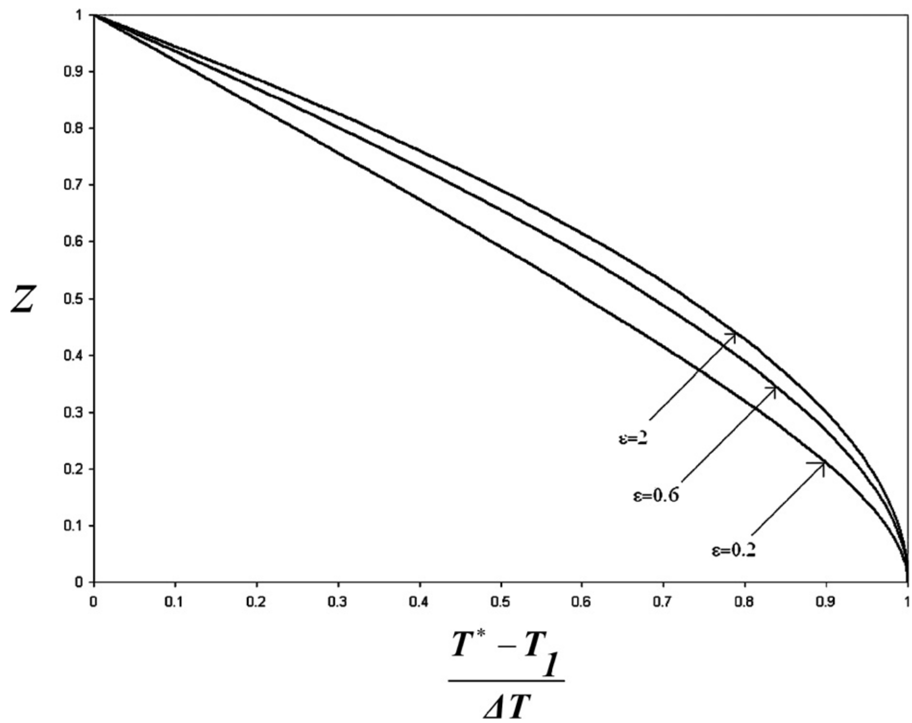


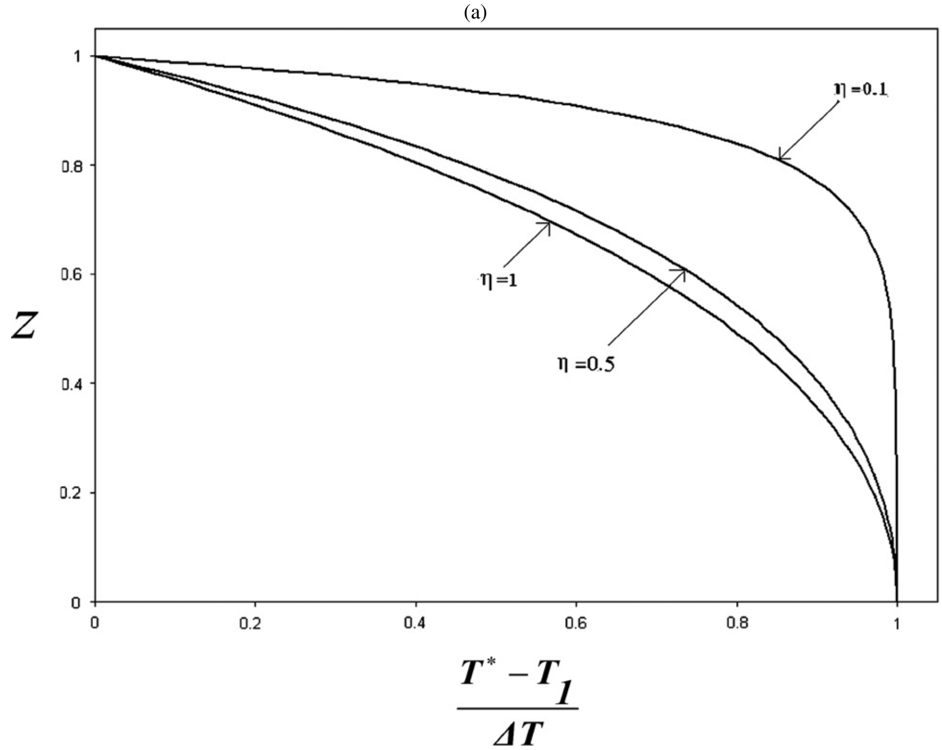
Fig. 4. Slope of the R_I–Ta curve ($\frac{dR_{IC}}{dT_a}$) plotted against the exponents, showing the effect of heat source distribution on rotating convection.

5.2. Constant temperature boundary condition

In the earlier section, the heat transfer at the bottom boundary was neglected. In this section, we consider the situation where both boundaries are at constant temperatures. Microwave ovens with both the top and the bottom surfaces exposed to the atmosphere can be considered to be a realistic application of this. It may also be noted that since we have imposed a boundary condition of temperature T_1 at both the boundaries, the onset of convection, here is thus, purely through internal heat generation, and there is no effect of wall heating, as such. Thus, the effect of internal heating, independent of the effects of wall heating on the convective stability has been studied.



(a)



(b)

Fig. 5. Temperature profiles for the two cases (a) exponential heat flux from the bottom, (b) exponential heat flux from the top.

5.2.1. Exponentially decaying heat source from the bottom

When exponential heat source is applied from the bottom, a solution of the mean state energy equation leads to the following temperature distribution:

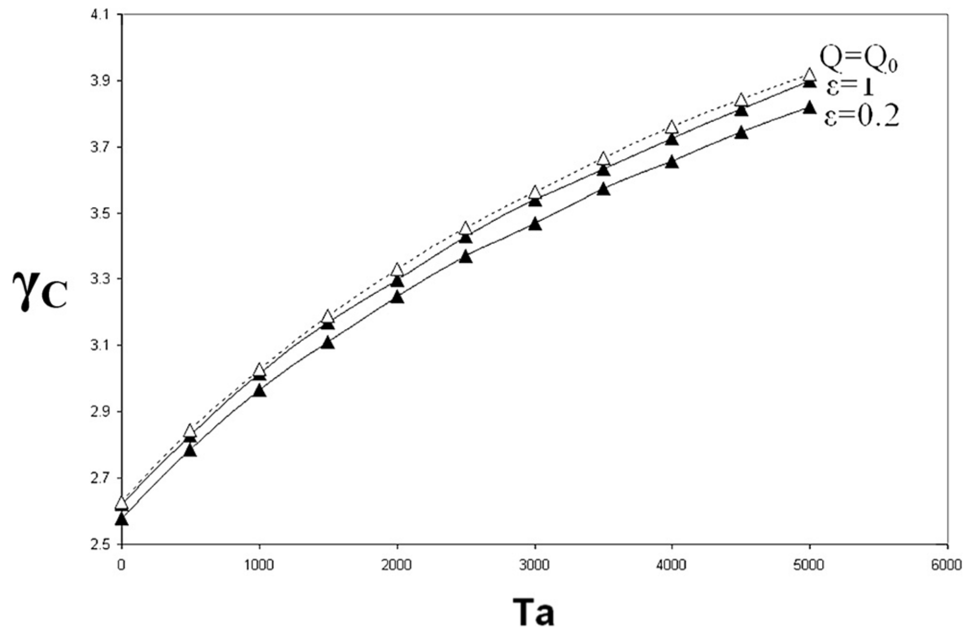
$$\frac{T^* - T_1}{\Delta T} = \frac{1}{G(\varepsilon)}\left[z\exp\left(\frac{-1}{\varepsilon}\right) - \exp\left(\frac{-z}{\varepsilon}\right) + 1 - z\right] \qquad (23)$$

where

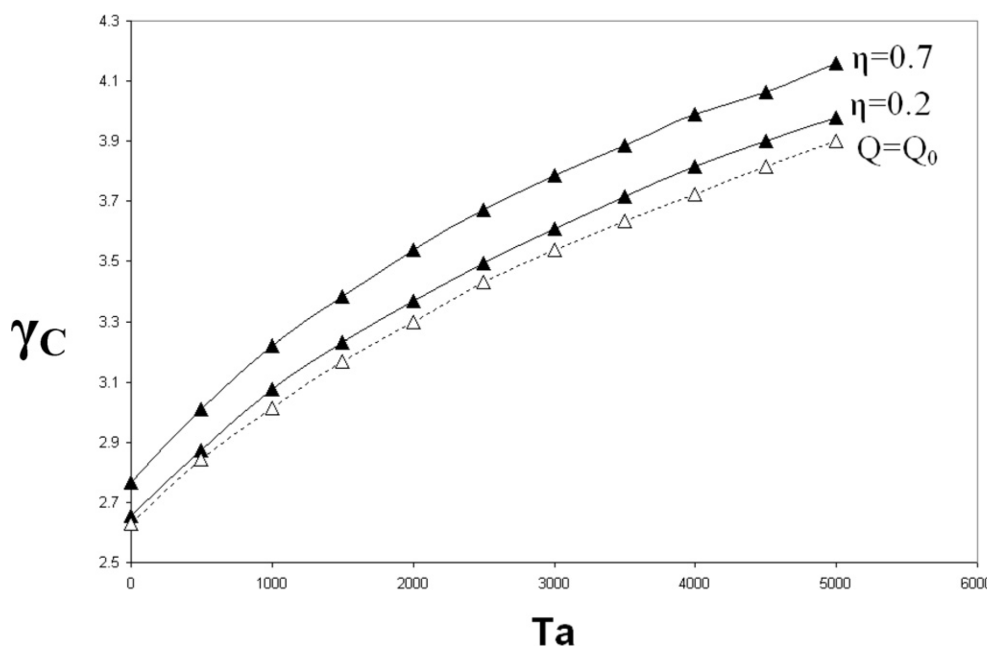
$$G(\varepsilon) = 1 - \left(1 + \frac{z'}{\varepsilon}\right)\exp\left(\frac{-z'}{\varepsilon}\right) \qquad (24)$$

Here, z' is the height where the maximum temperature is reached. The expression for z' reads:

$$z' = -\varepsilon\left[\ln\varepsilon + \ln\left\{1 - \exp\left(-\frac{1}{\varepsilon}\right)\right\}\right] \qquad (25)$$



(a)



(b)

Fig. 6. (a) The variation of critical wave number with Taylor number for different ε. (b) The variation of Critical Wave number with Taylor number for different η.

The maximum temperature variation, used to non-dimensionalize the temperature, is given as:

$$\Delta T = \frac{Q_0 L^2 \varepsilon^2}{\lambda H(\varepsilon)} G(\varepsilon)$$

The temperature profile is shown in Fig. 7, for different values of ε, and the shape of the distribution for uniform heat source distribution is given as a comparison. A stable layer is seen to form in this case, the consequences of which shall be elaborated on later.

This mean temperature distribution gives the following equation for perturbation:

$$(D^2 - \gamma^2)\hat{T} = \frac{-1}{G(\varepsilon)}\left[\frac{1}{\varepsilon}\exp\left(\frac{-z}{\varepsilon}\right) - 1 + \exp\left(\frac{-1}{\varepsilon}\right)\right]\hat{w} \quad (26)$$

In this case, the internal Rayleigh number is defined as:

$$R_I = \frac{g\beta Q_0 L^5 \eta^2}{\nu \kappa_0 \lambda H(\varepsilon)} G(\varepsilon)$$

The variation of critical internal Rayleigh number with ε is shown in Fig. 8 for different values of Taylor number. The values show the same general trend, however with increased values of the critical Rayleigh number. The critical Rayleigh number for $\varepsilon = 0.2$ is 1771, which is more than the critical Rayleigh number for simple Rayleigh Benard convection (1707.8). While

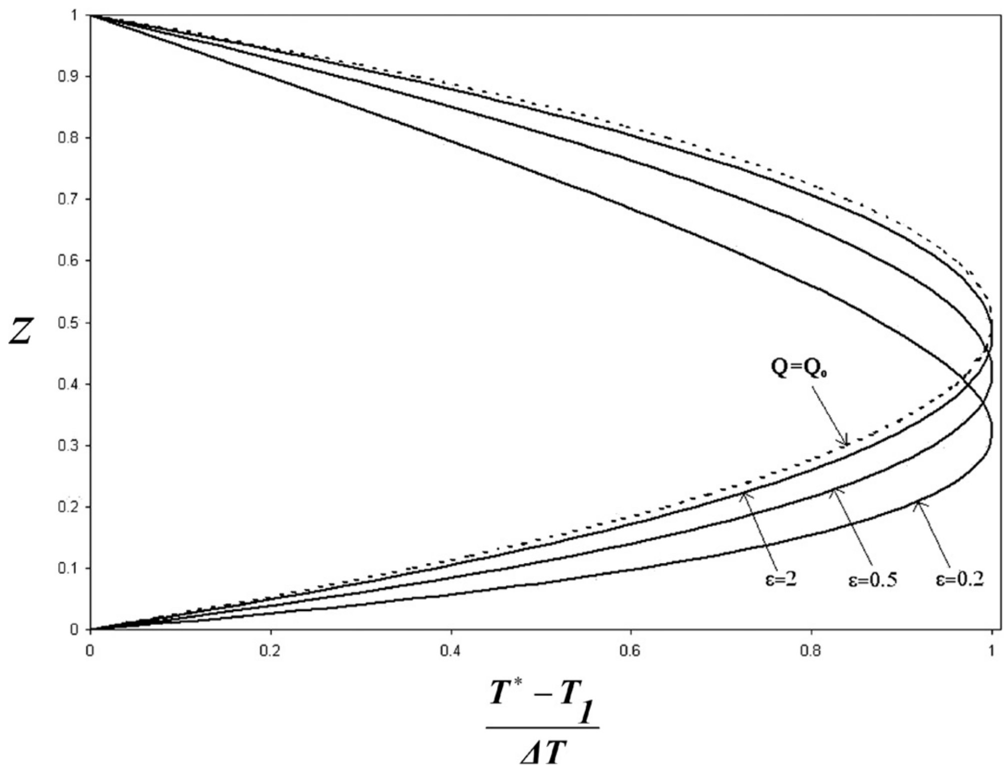


Fig. 7. Temperature profiles when heat flux is applied from the bottom, and the same constant temperature is maintained at both the surfaces.

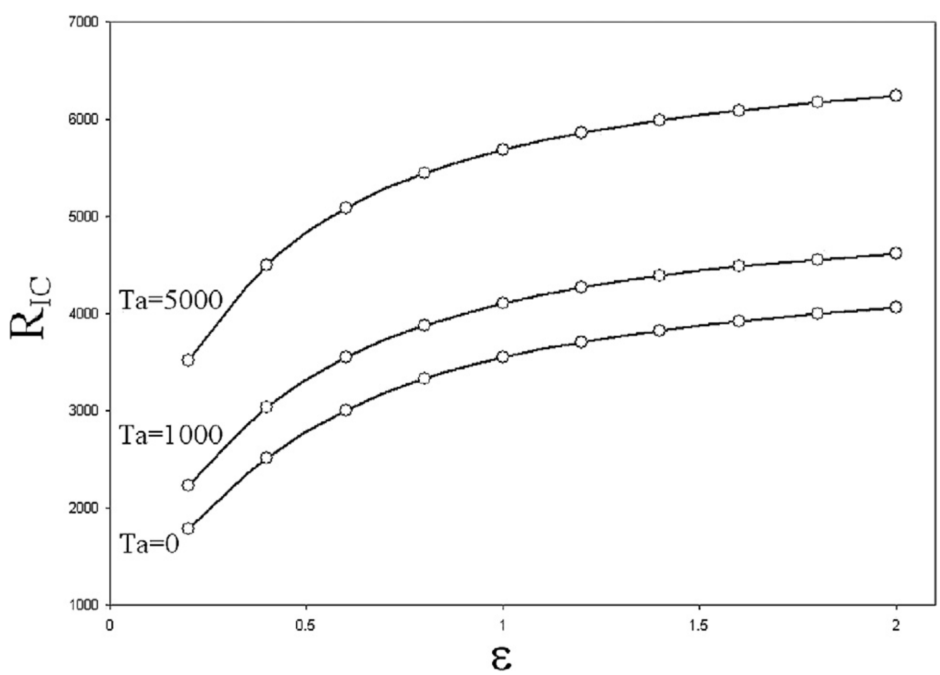


Fig. 8. Evolution of Rayleigh numbers with ε in the constant temperature boundary condition case. Heat source is concentrated at the bottom.

on the other hand, in the case of adiabatic boundaries, an exponent of 0.2 produces R_I much lower than that of simple RBC.

Thus it can be seen that the advantages of using an internal heat generation is dependent to a large extent on the boundary conditions used. In the limit of a constant temperature boundary condition, Rayleigh Benard convection seems to be less probable.

5.2.2. Exponentially decaying heat source from the top

This configuration of the heat source gives rise to the following temperature profile:

$$\frac{T^* - T_1}{\Delta T} = \frac{1}{G(\eta)}\left[\exp\left(\frac{z-1}{\eta}\right) - z + (z-1)\exp\left(\frac{-1}{\eta}\right)\right] \tag{27}$$

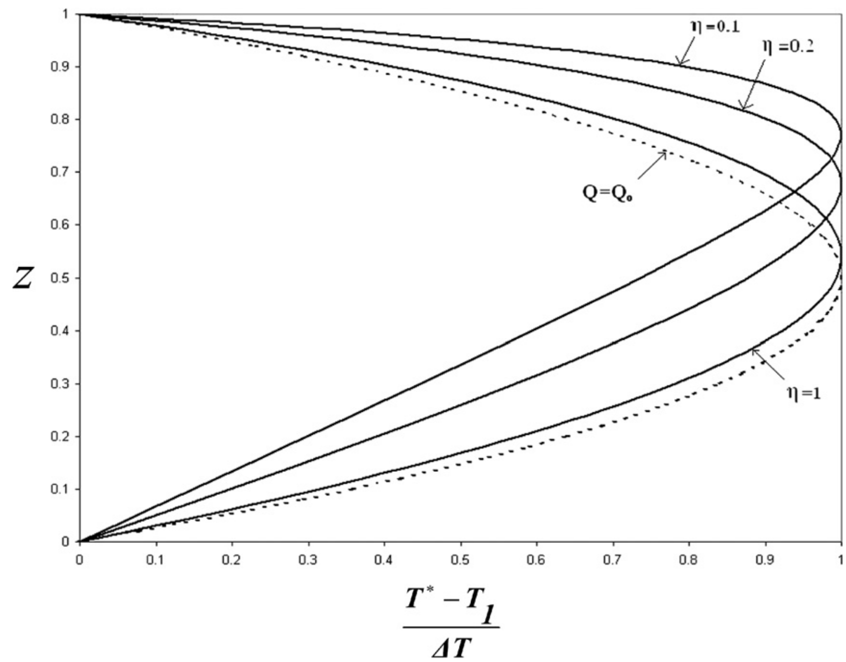


Fig. 9. Temperature profiles when heat flux is applied from the top, and the boundary condition is of constant temperature.

where

$$G(\eta) = \left(1 - \frac{z'}{\eta}\right)\exp\left(\frac{z'-1}{\eta}\right) - \exp\left(\frac{-1}{\eta}\right) \tag{28}$$

In this case

$$z' = \eta \ln \eta + \eta \ln\left[1 - \exp\left(\frac{-1}{\eta}\right)\right] + 1 \tag{29}$$

The shape of the temperature profile is plotted in Fig. 9 and the maximum temperature difference

$$\Delta T = \frac{Q_0 L^2 \eta^2}{\lambda H(\eta)} G(\eta)$$

The expression for the internal Rayleigh number is:

$$R_I = \frac{g\beta Q_0 L^5 \eta^2}{\nu \kappa_0 \lambda H(\eta)} G(\eta)$$

Finally, we derive the perturbation equation for the temperature as:

$$(D^2 - \gamma^2)\hat{T} = \frac{-1}{G(\eta)}\left[\frac{1}{\eta}\exp\left(\frac{z-1}{\eta}\right) - 1 + \exp\left(\frac{-1}{\eta}\right)\right]\hat{w} \tag{30}$$

The above equation is solved for different values of the Taylor number, and the results are plotted in Fig. 10. An asymptotic rise at small values of η leads to exceptionally high values of R_{IC}.

5.2.3. Effect of rotation

The base temperature profiles in both cases are seen to give rise to stable layers with positive temperature gradients. This is one of the fundamental differences of this case with the previous adiabatic case, where there was, at best quasi-stable layers.

The slope of the R_{IC}–Ta curve has been plotted with the exponents as before (Fig. 11). However, in this case, we see a marked difference from the previous case. Rotation seems to be sensitive to both cases of exponential profiles, and more or less to the same extent. In this context, it may be noted that quasi-stable layer in the temperature profiles for both cases of heat source distribution exist, as shown in Figs. 7 and 9. Hence, one can extend the previous argument to note that stable layers with positive temperature gradients enhance the stabilizing effect of rotation.

Fig. 12 shows the change in critical wave numbers with rotation rates. The general trend of increase in the size of the convection cell with rotation is observed here also. Also, note that while in the adiabatic case, there were only slight changes in wave number with ε (while there was a large change in size with change in η) [12], in this case, both the changes in ε and η cause a significant change in the wave number, at all rotation rates (20.13% for change in ε, and 47.13% for change in η, in the static case). However, heat source concentration at the top causes a much greater increase in the wave number this case also, as before.

6. Conclusion

A systematic study of the stability criterion of rotating horizontal fluid layers with internal heat generation has been carried out for various boundary conditions and heat source orientations, using the Chebyshev pseudospectral methods. The results have been presented in terms of critical Rayleigh number and wave number. From this wide ranging study, it can be concluded that:

1. The R_{IC}–Ta relationship is linear for moderate values of Taylor number, irrespective of the orientation of the heat

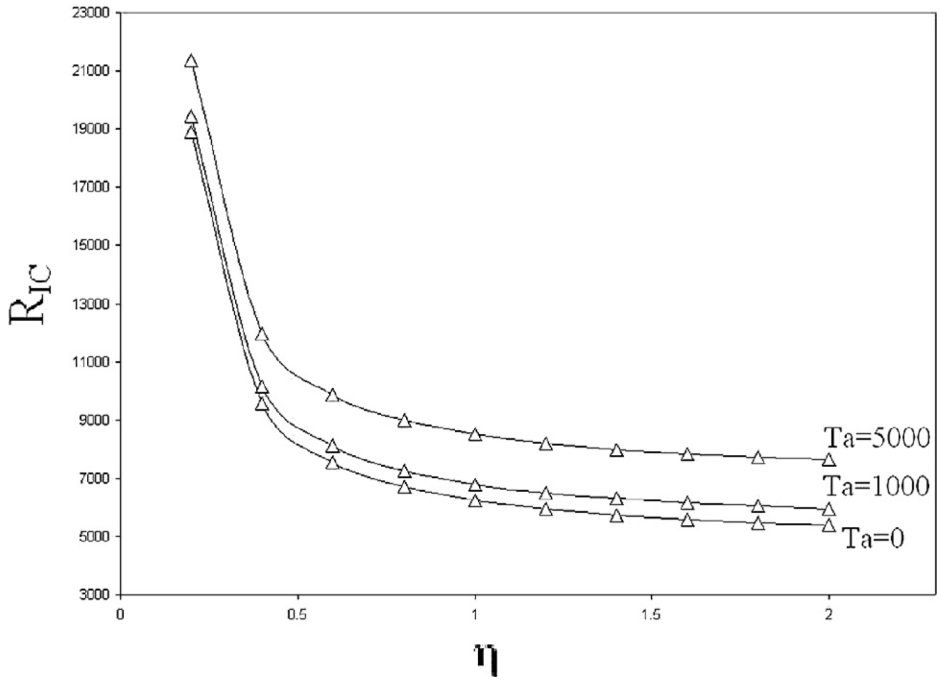


Fig. 10. Rayleigh number variation with η. In this case, the exponentially decaying heat source is applied from the bottom.

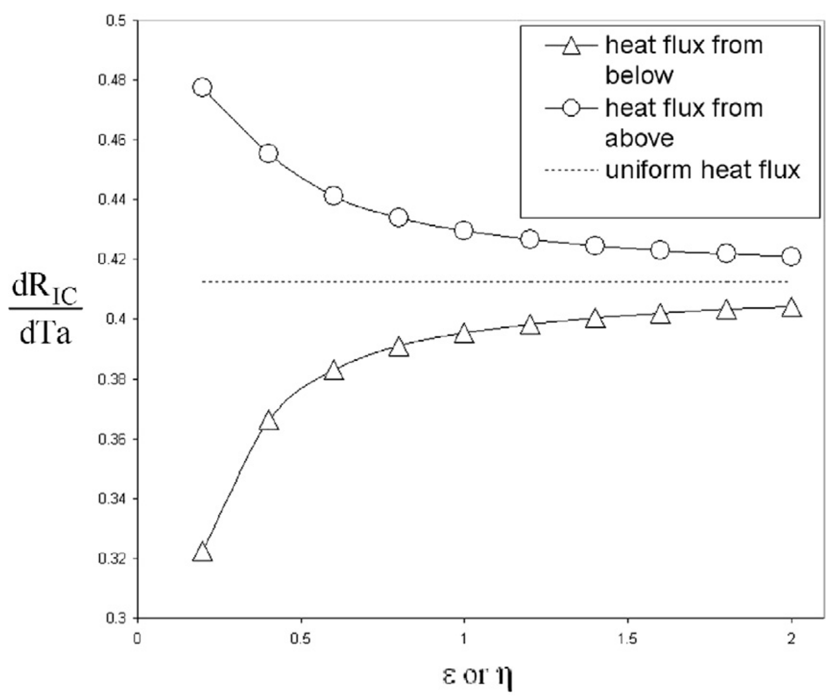


Fig. 11. Effect of heat source distributions on rotation rates in case of constant temperature boundary conditions.

source distribution or the imposed thermal boundary condition.

2. The effect of rotation on the stability of the system is highly sensitive to the orientation of heat source distribution in case of stable and quasi stable temperature profiles.
3. In cases where the resulting temperature profiles do not consist of stable or quasi stable layers, the effect of rotation is merely to stabilize the temperature profile, to a similar extent for various heat source configurations.

With the present study, it has been possible to clarify the coupled effects of rotation and internal heat generation in affecting the stability of thermal convection. Keeping in mind, the extensive uses of such convection processes, especially in the food and pharmaceutical industry, this study would go a long way in deciding the different operating parameters in such processes, such as rotation, the strength and the orientation of the heat source.

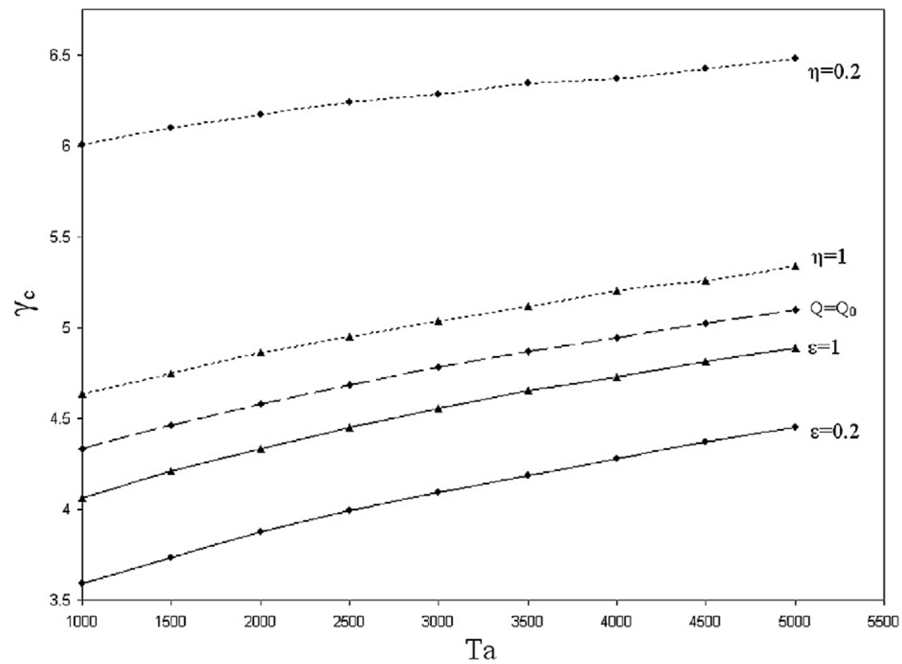


Fig. 12. The variation of critical wave number with Taylor number for different ε or η. The boundary condition imposed is a constant temperature at the ends.

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